

$$\mathbf{F} = e(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}) \dots \text{Force on a charged particle}$$

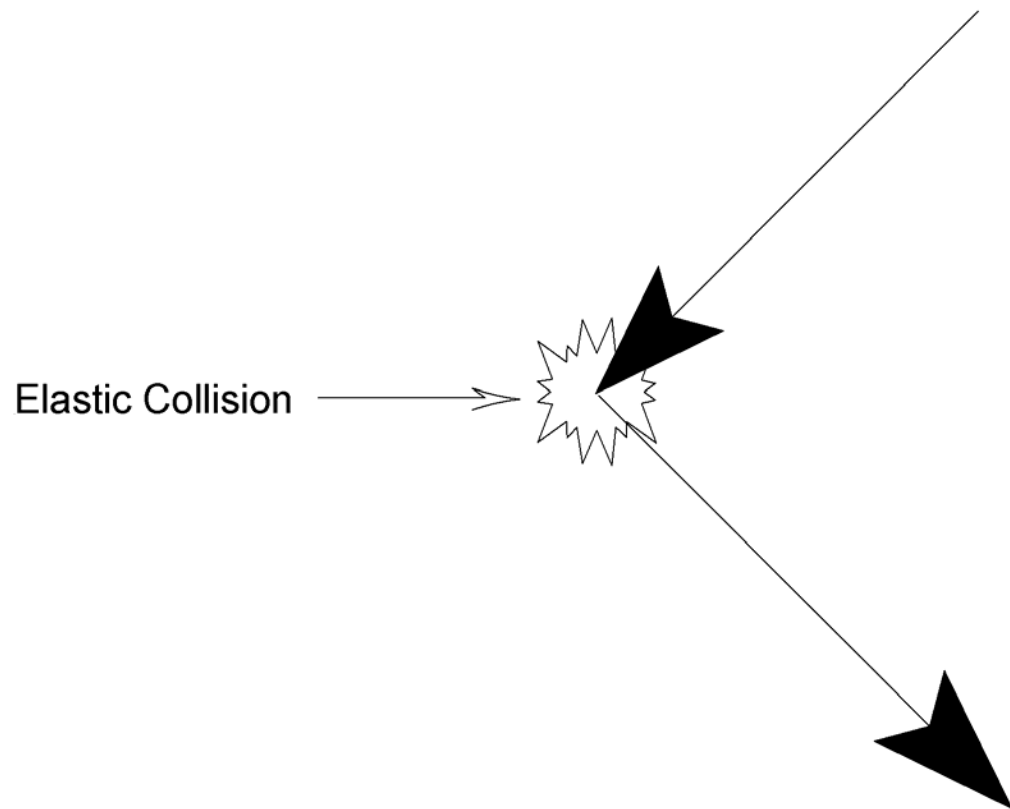
$$\mathbf{F} \cdot \mathbf{v} = e\mathbf{E} \cdot \mathbf{v} \dots \text{Work done on a charged particle}$$

In a conducting plasma at rest $\mathbf{E} = 0$ but in a frame where the plasma moves with velocity \mathbf{U} we have:

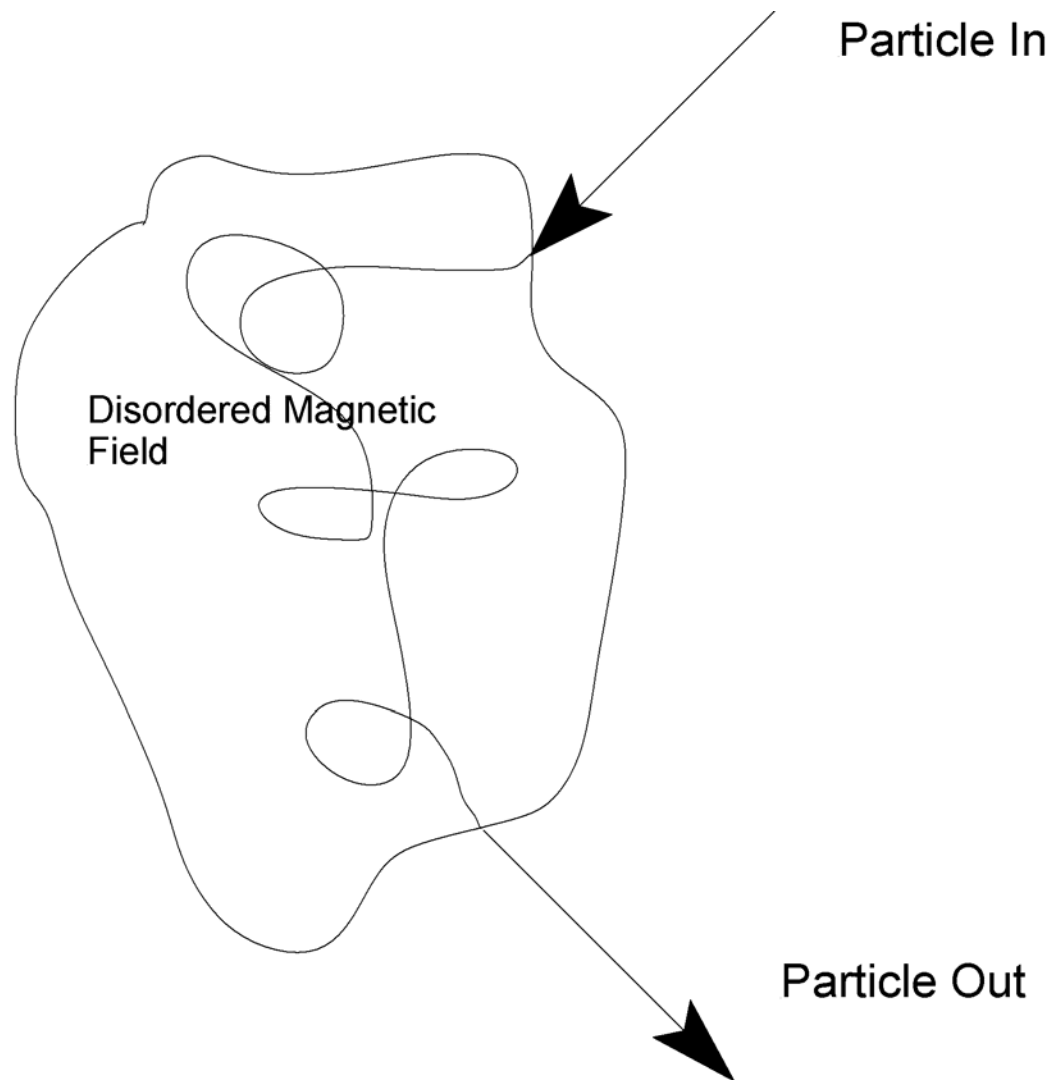
$$\mathbf{E} = -\frac{\mathbf{U}}{c} \times \mathbf{B}$$

and

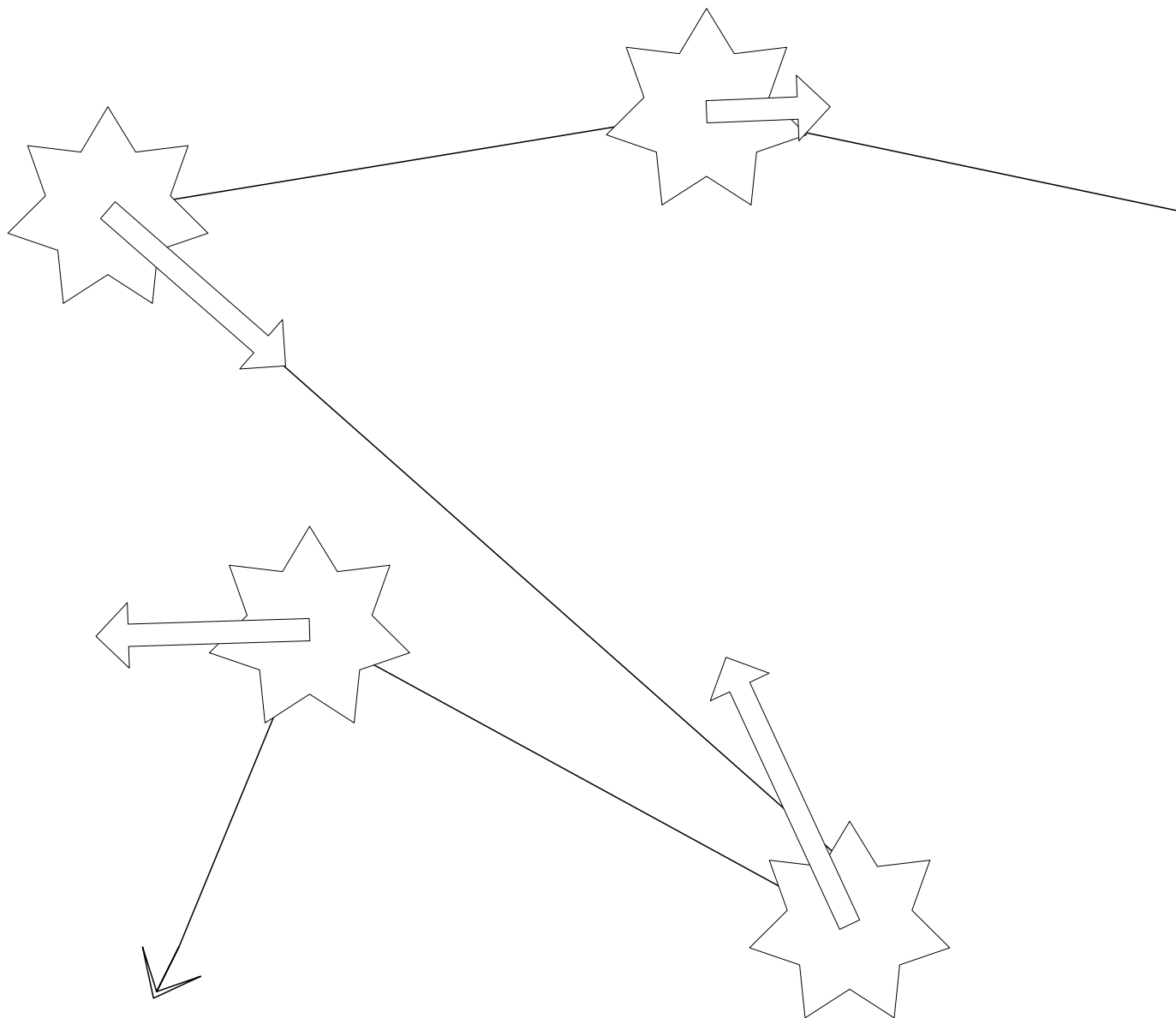
$$\begin{aligned} \mathbf{F} \cdot \mathbf{v} &= -e \left(\frac{\mathbf{U}}{c} \times \mathbf{B} \right) \cdot \mathbf{v} \\ &= \mathbf{U} \cdot \left(e \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \\ &= \frac{d\mathcal{E}}{dt} \dots \text{particle} \end{aligned}$$



Particle scattering.



Particle scattering in disordered magnetic field in a plasma cloud.



$$\frac{1}{p} \frac{dp}{dt} = \nu (U/c)^2 \equiv \frac{1}{\tau}$$

$$\frac{1}{n} \frac{dn}{dt} = -(1/T)$$

$$\Downarrow$$

$$\ln(p) = \frac{t}{\tau}$$

$$\ln(n) = -t/T$$

$$\Downarrow$$

$$\ln(n) = \left(\frac{-\tau}{T}\right) \ln(p)$$

$$\Downarrow$$

$$n(p) = p^{\frac{-\tau}{T}}$$

$$\nu = \frac{c}{\lambda}$$

COLLISION RATE

$$\tau = \frac{1}{\nu} \frac{c^2}{U^2} = \frac{\lambda c}{U^2}$$

ENERGY GAIN TIME

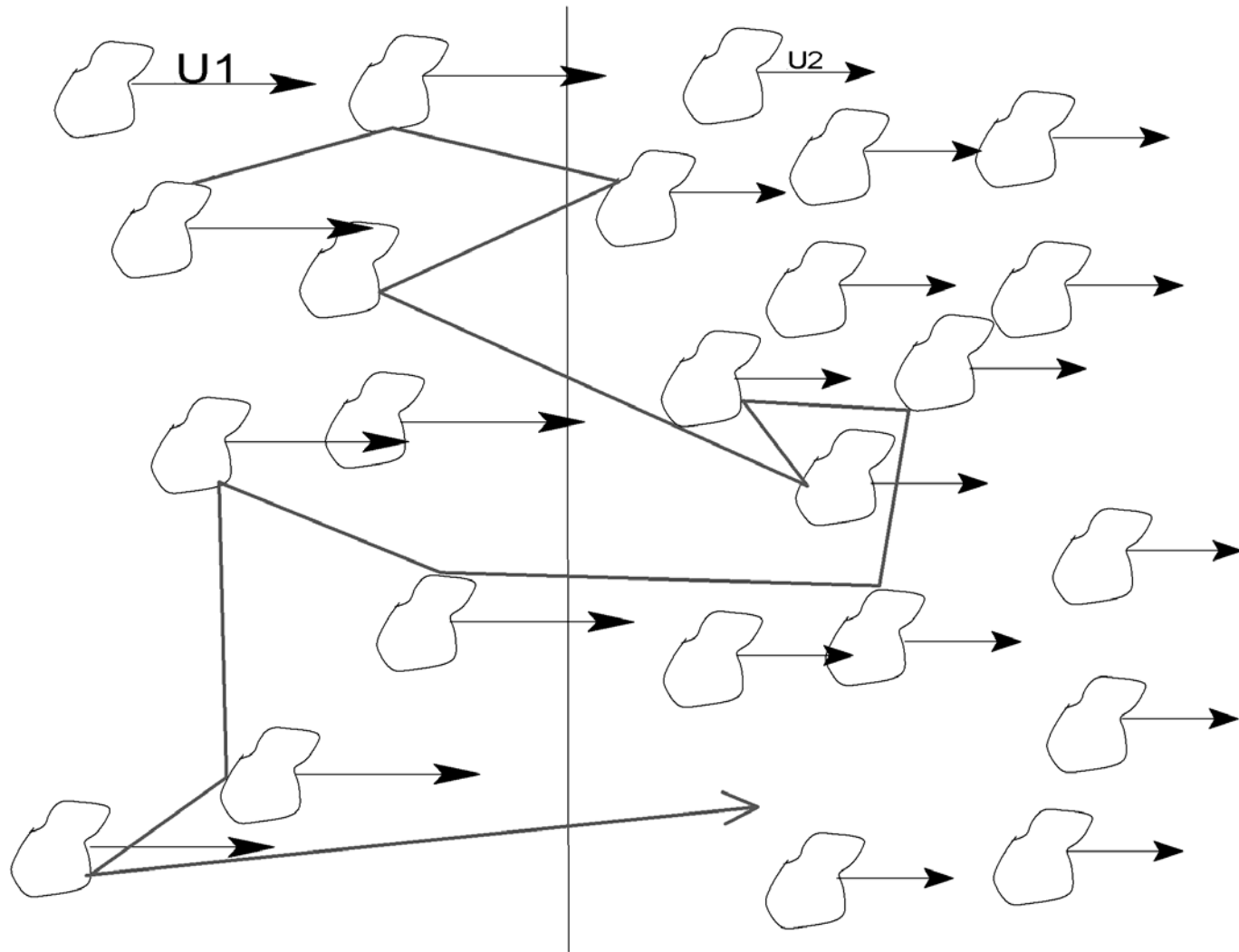
$$\frac{1}{T} = \frac{c\lambda}{3L^2}$$

ESCAPE TIME

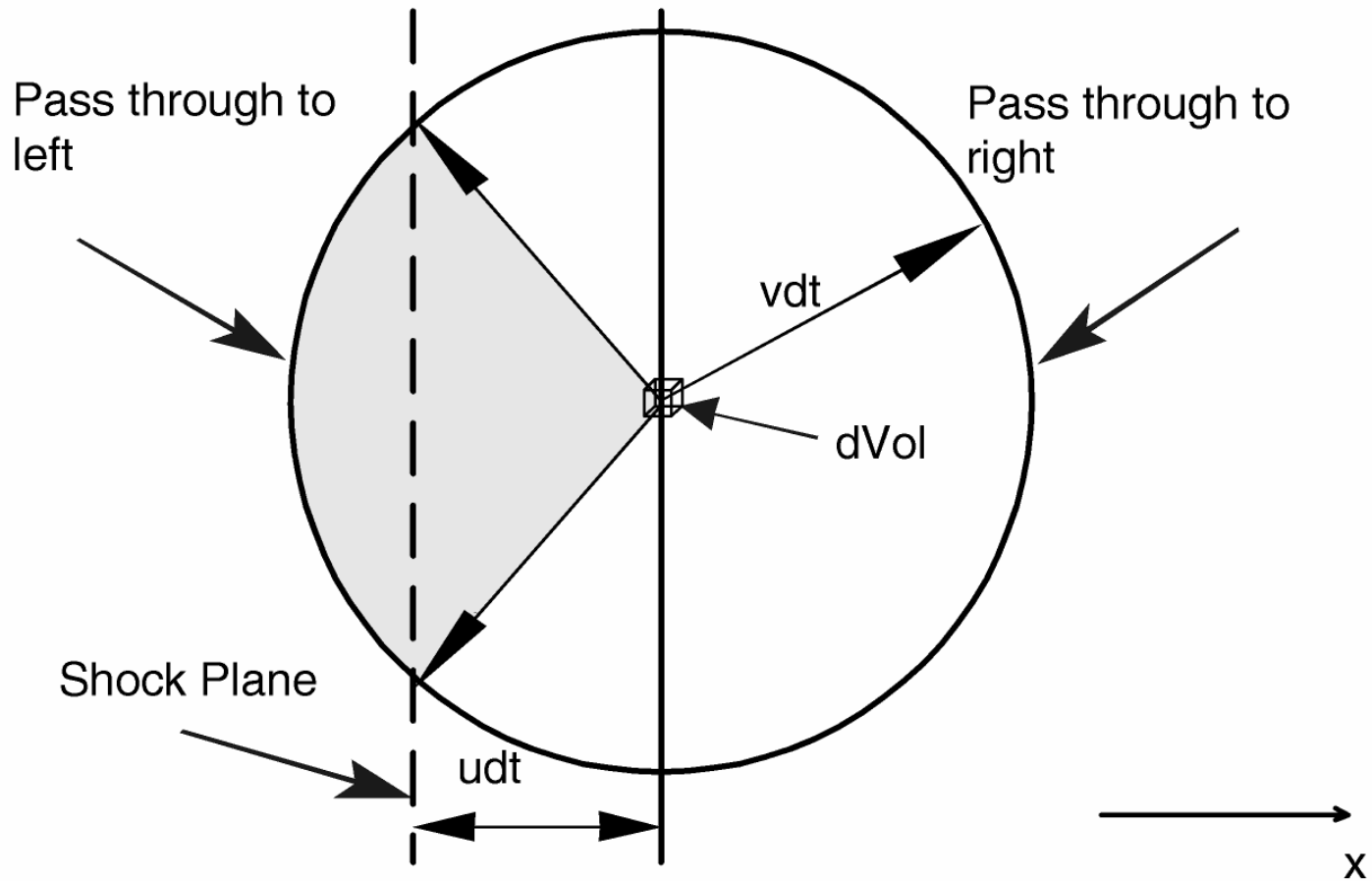
$$\frac{\tau}{T} = \frac{c^2 \lambda^2}{U^2 L^2}$$

EXPONENT

Sir Charles Darwin – 1949, *Nature* **164**, 1112

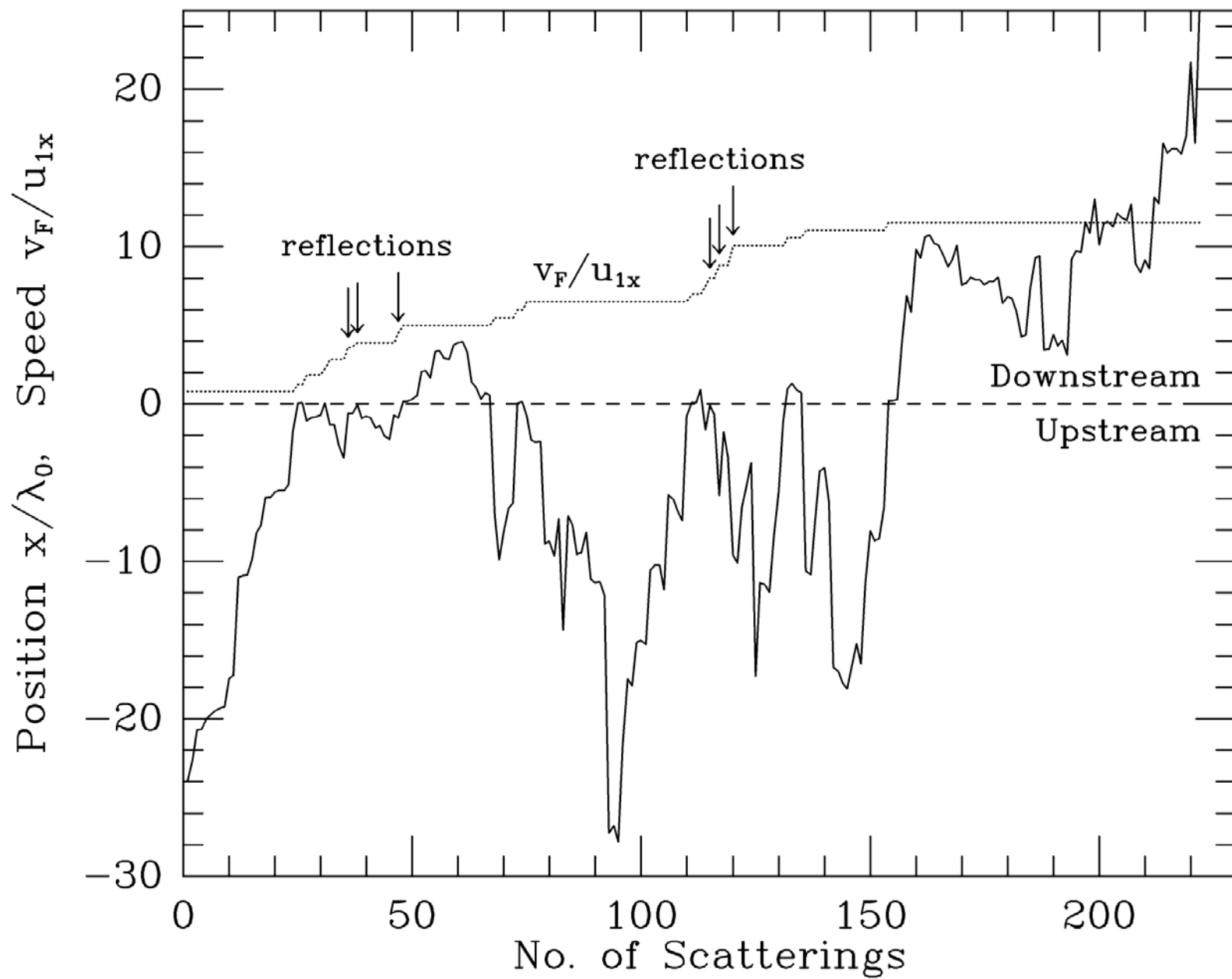


Particle scattering back and forth across a shock.



Return Probability from velocity space. The circle shown is the projection on the $x - z$ plane of a sphere with radius vdt , a portion of which protrudes through the shock plane.

$$\Theta_{B1} = 25^\circ, \mathcal{M}_S = 2.7, r = 2.83$$



$$\frac{1}{p} \frac{dp}{dN} = \frac{4}{3} \frac{U_1 - U_2}{c}$$

$$\frac{1}{n} \frac{dn}{dN} = -4 \frac{U_2}{c}$$

$$\Downarrow$$

$$\ln(p) = N \frac{4}{3} \frac{U_1 - U_2}{c}$$

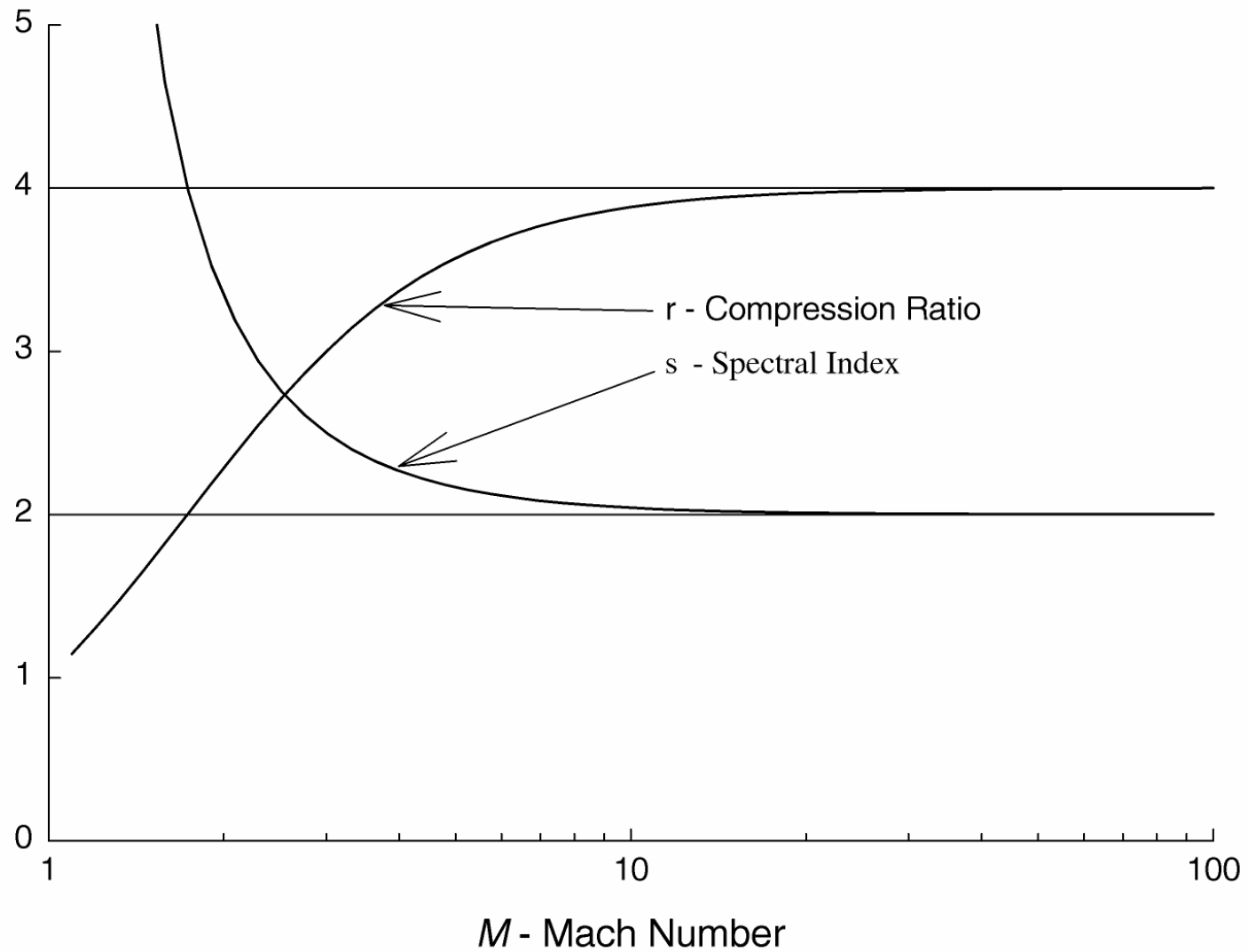
$$\ln(n) = -N \frac{4}{c} U_2$$

$$\Downarrow$$

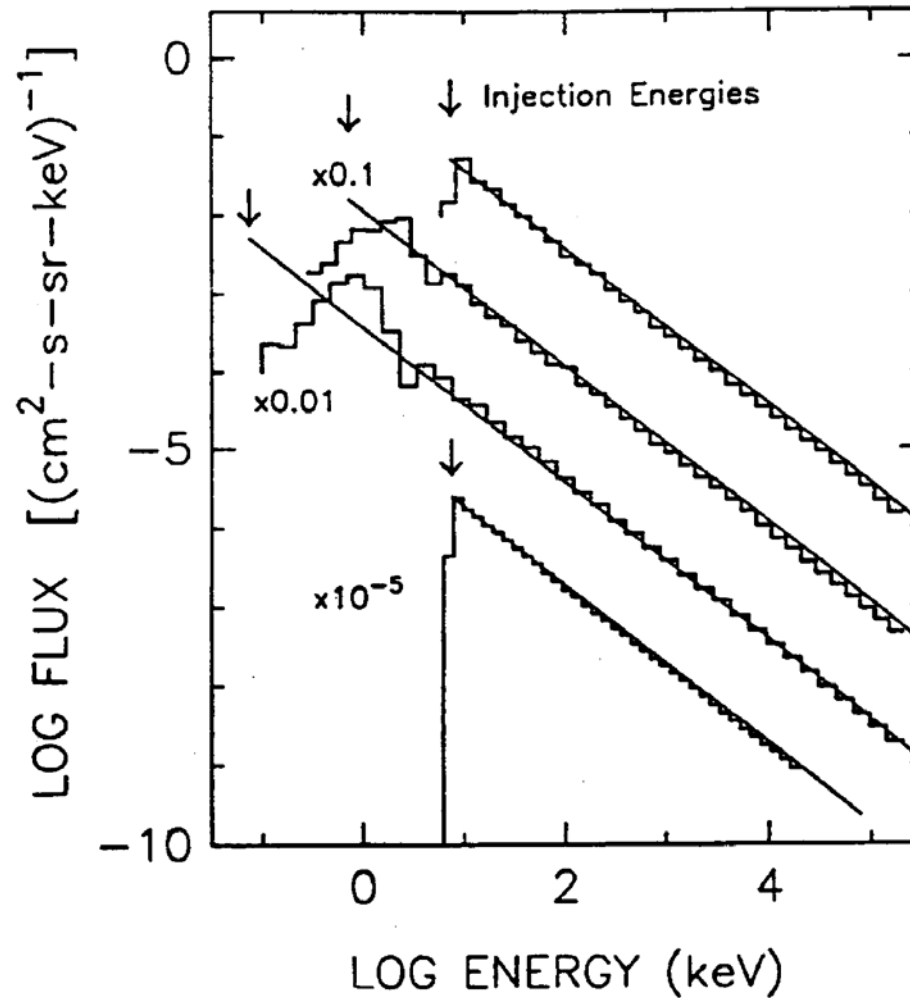
$$\ln(n) = \left(\frac{-3}{U_1 - U_2} U_2 \right) \ln(p)$$

$$\Downarrow$$

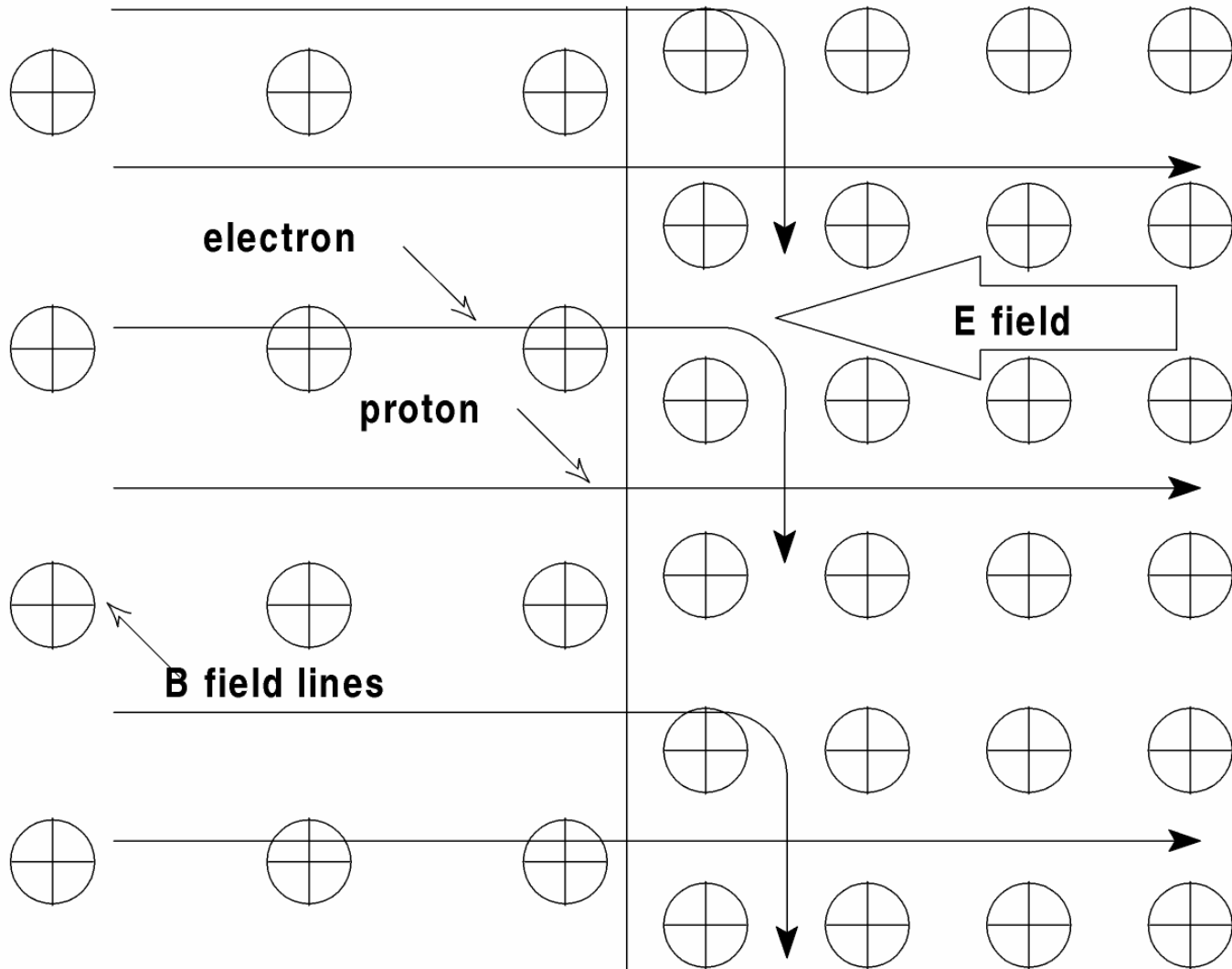
$$n(p) = p^{\frac{-3}{r-1}} \quad (r = U_1/U_2)$$



The compression ratio and spectral index produced by a shock as a function of the incoming Mach number.



Particle flux versus energy for nonrelativistic shock velocities. The upper six curves were calculated with $u_1 = 500 \text{ km s}^{-1}$, $r = 4$, and $\alpha = 1$ (see equation 92). The lower two curves were calculated with $u_1 = 100 \text{ km s}^{-1}$. The smooth curves are the test particle predictions (equation 22) while the histograms are the Monte Carlo results. All spectra here and elsewhere are calculated in the reference frame of the shock, at a downstream position, and particle fluxes are normalized to 1 incoming particle/($\text{cm}^2\text{-s}$). Injection energies are shown by arrows, and curves with numerical factors have been displaced by those factors for clarity. Figure from Ellison, Jones, and Reynolds (1990).



Schematic of a perpendicular shock

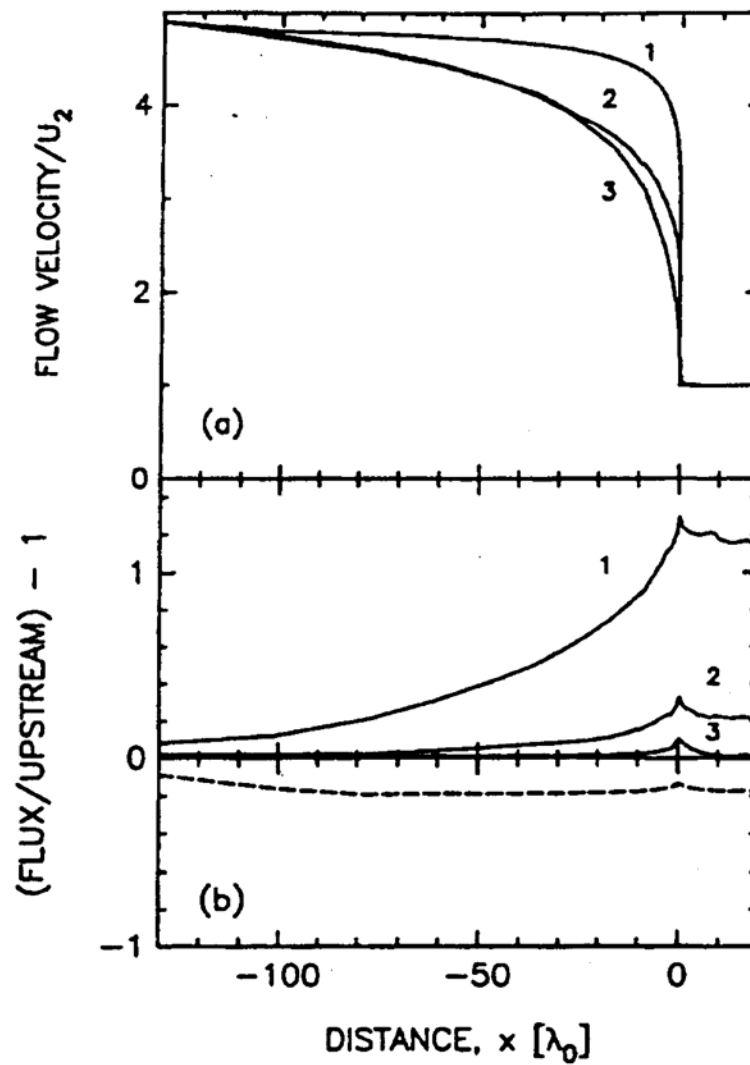


Fig. 13.— (a) Flow velocity versus distance as obtained from the Monte Carlo simulation. (b) Solid lines are the momentum flux and the dashed line is the energy flux at distances upstream and downstream from the abrupt subshock positioned at $x = 0$. Numbers indicate successive iterations. Fluxes are normalized so that the x -axis represents the far upstream value. Figure from Ellison, Möbius, and Paschmann (1990).